BLAST 2026

Baylor University

Small large cardinals and neostability theory

James E Hanson
Iowa State University

Indiscernible sequences

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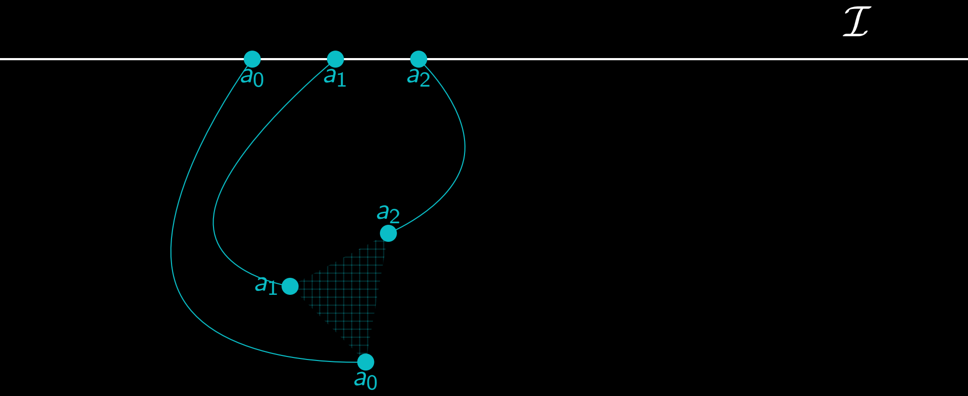
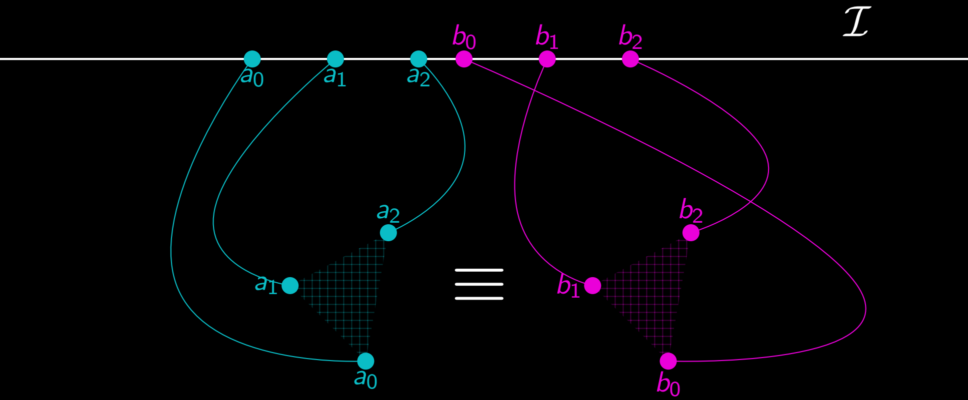
Indiscernible sequences

Indiscernible sequences



Indiscernible sequences



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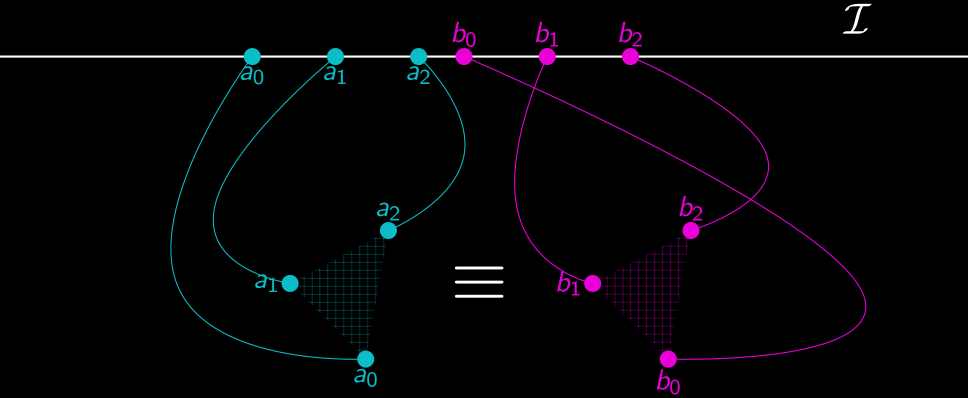
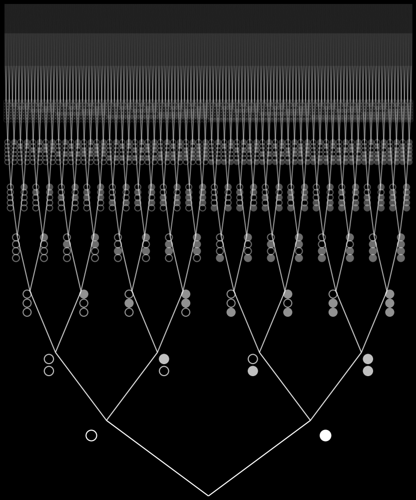
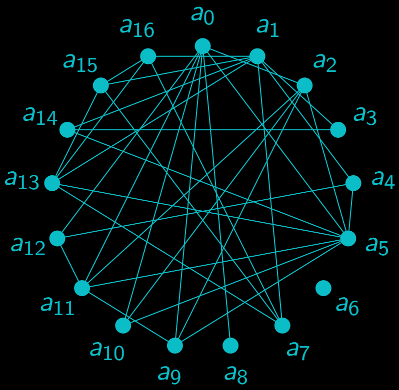


Ramsey's theorem + compactness $\rightarrow$ these exist in some model

A proof of Ramsey, Erdős–Rado





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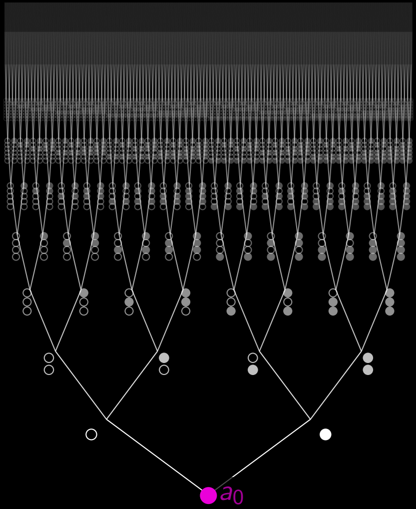

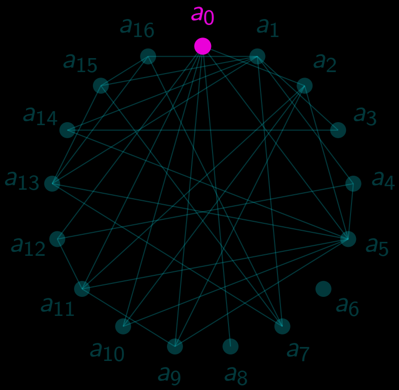

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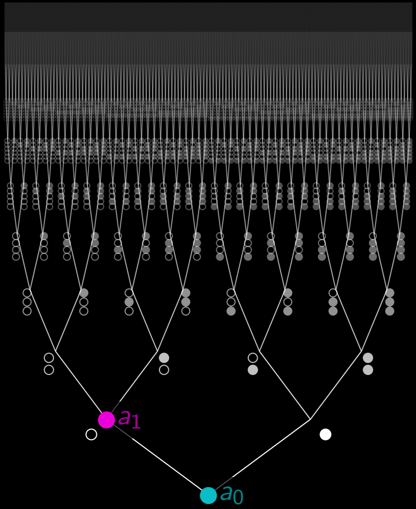

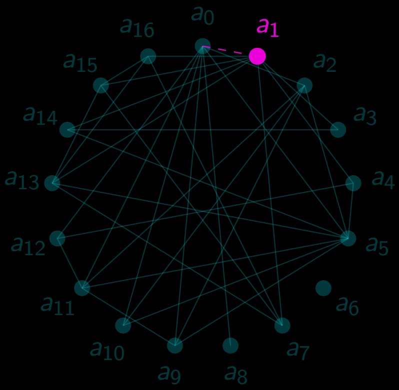

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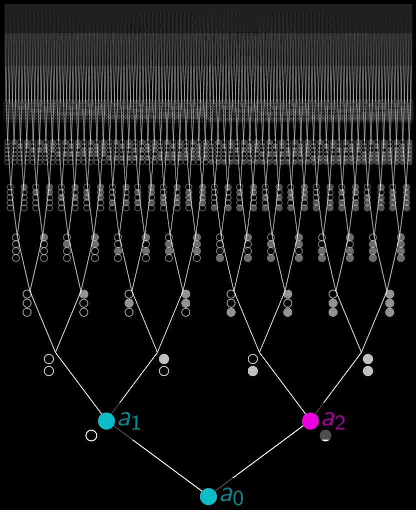

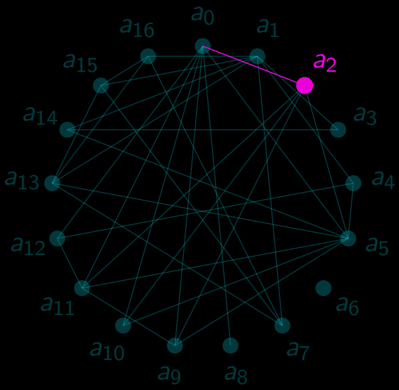

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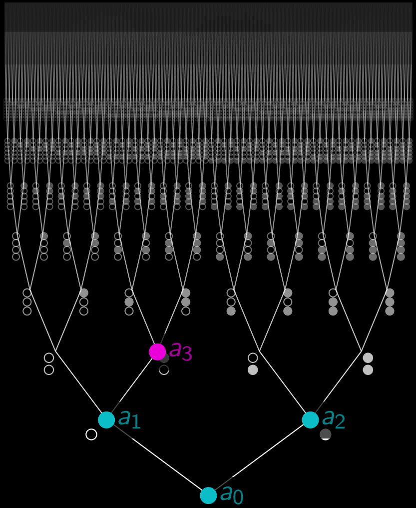

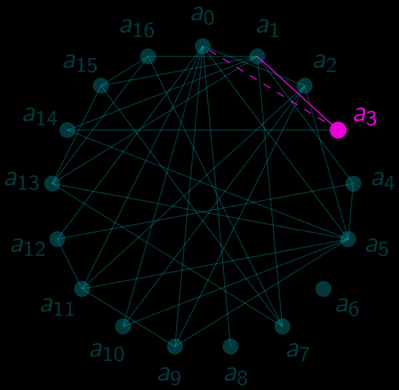

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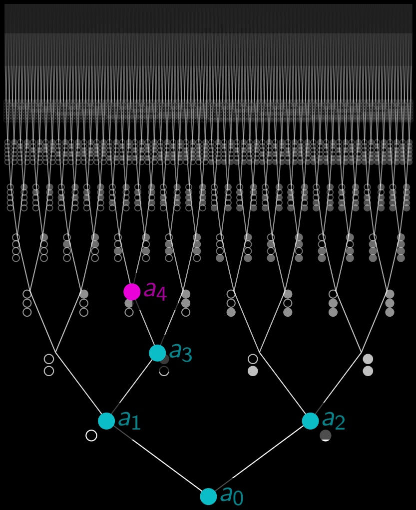

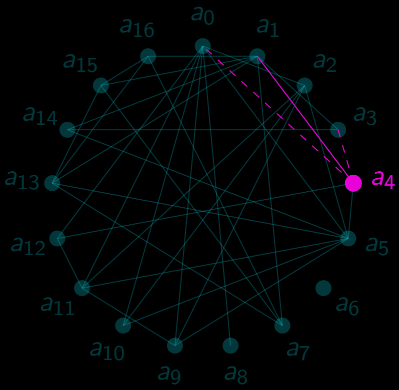

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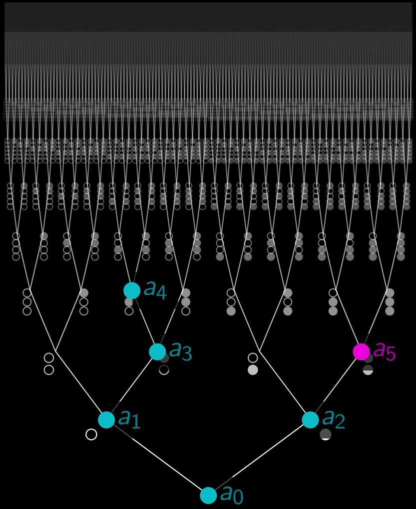

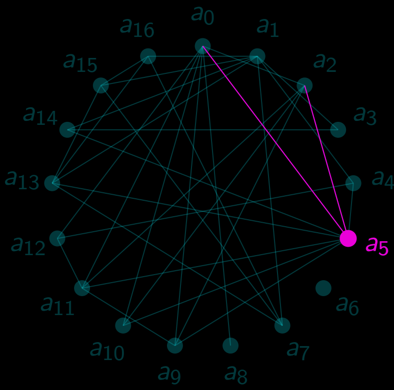

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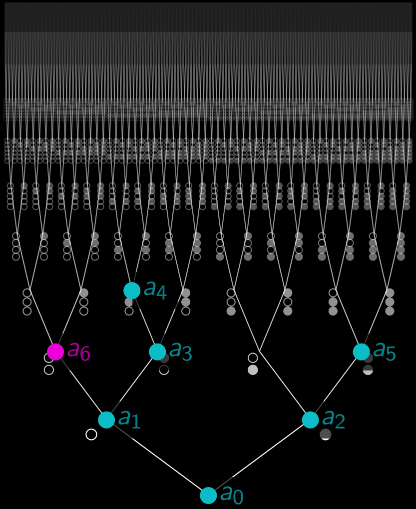

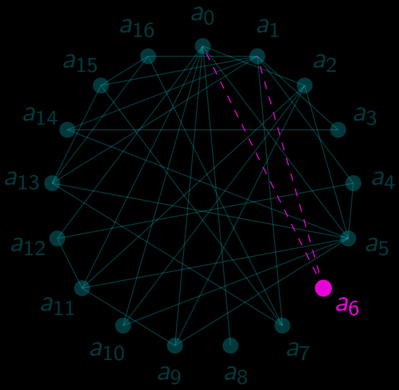

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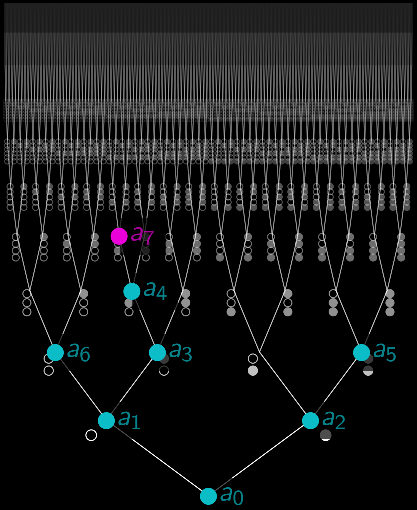

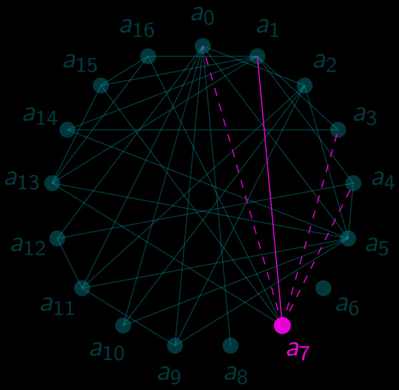

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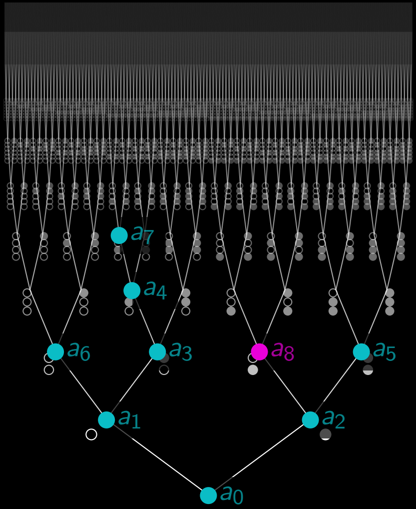

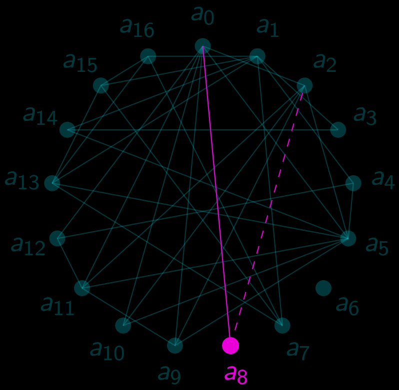

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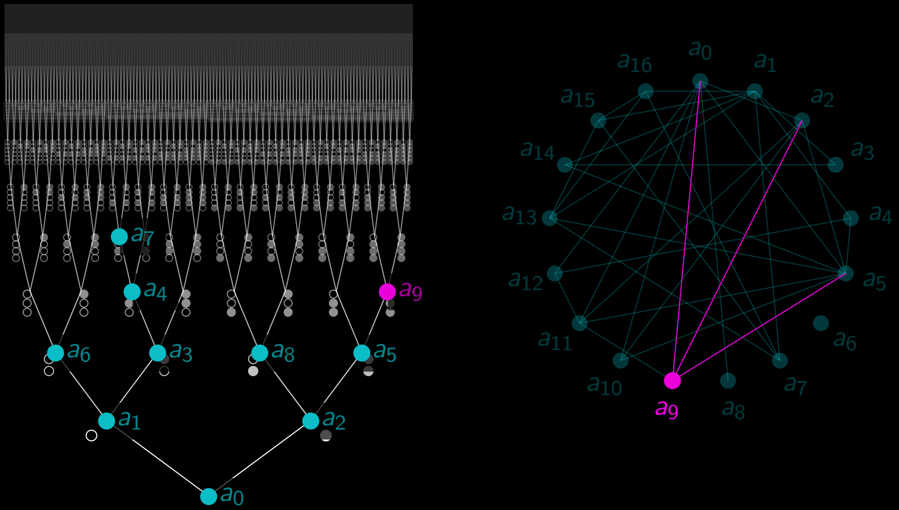

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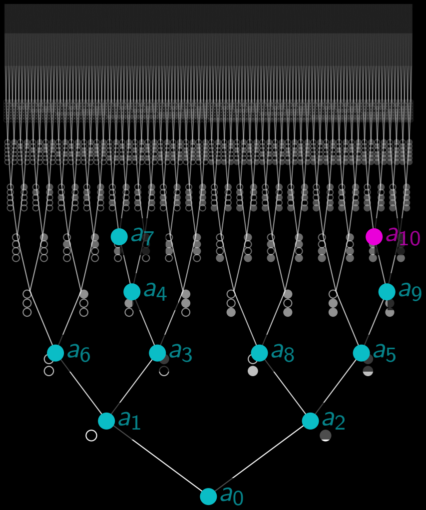

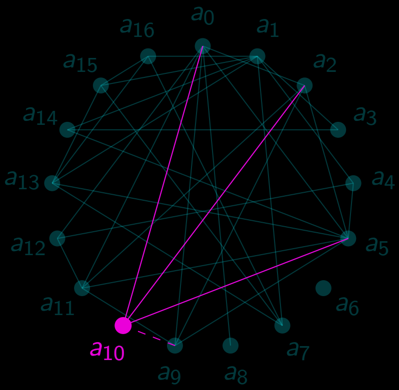

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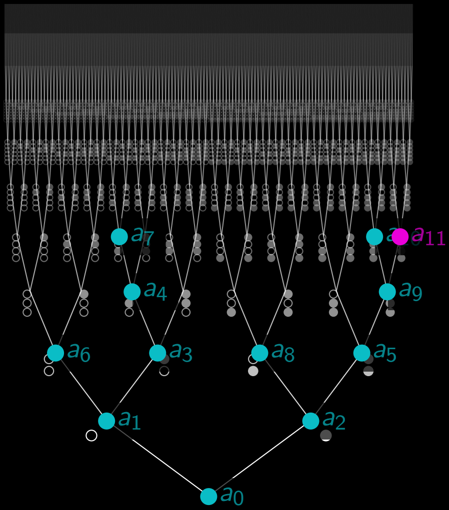

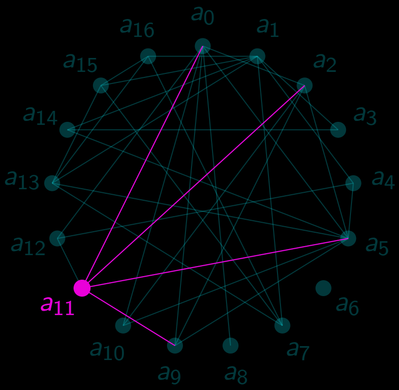

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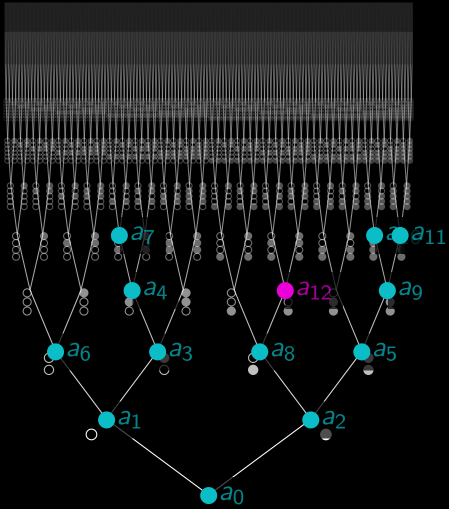

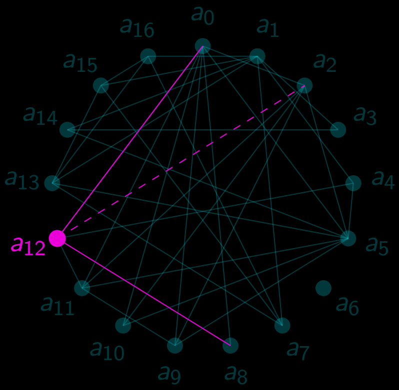

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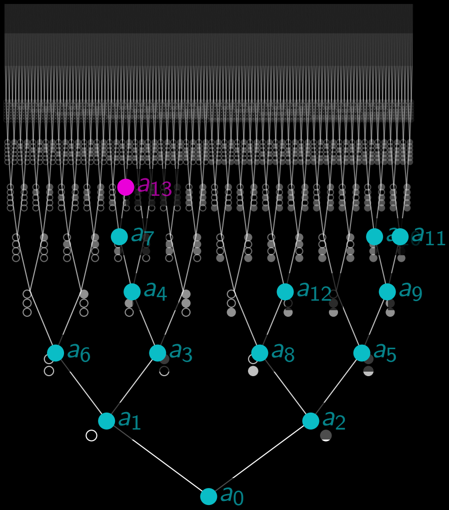

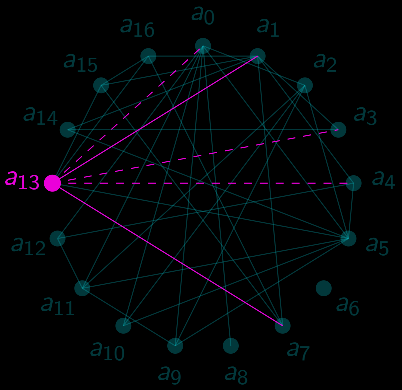

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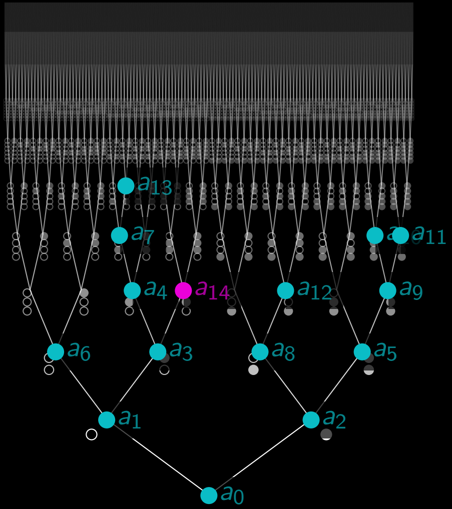

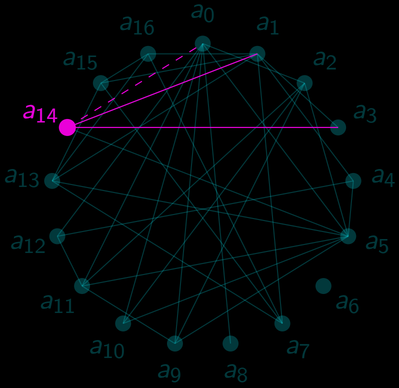

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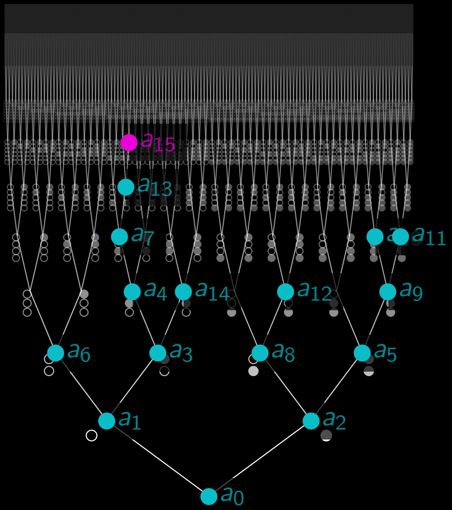

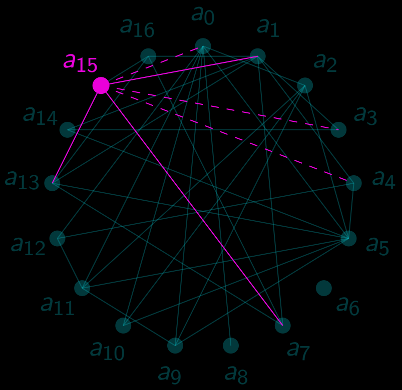

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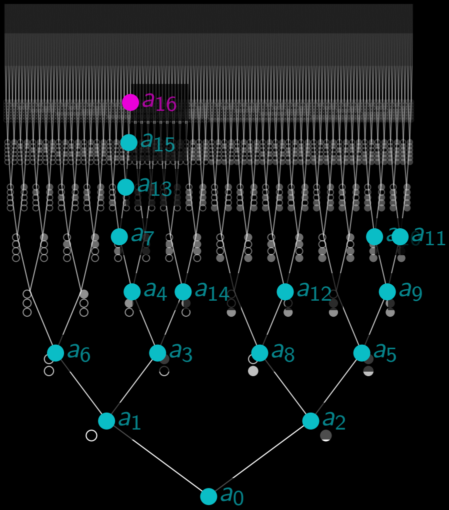
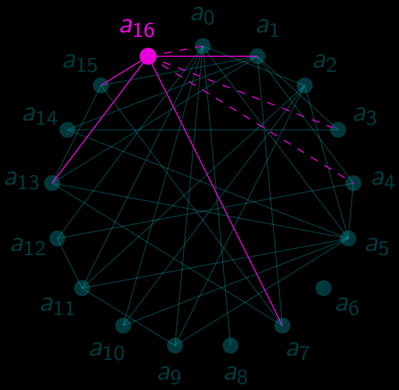
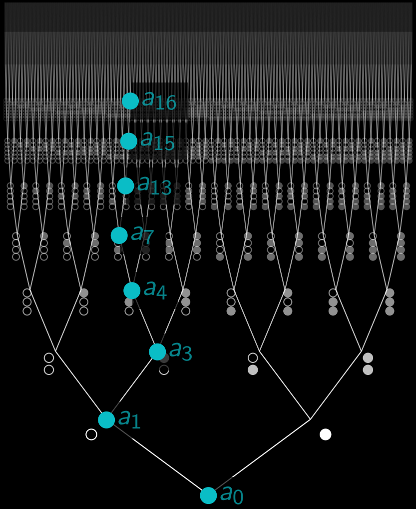
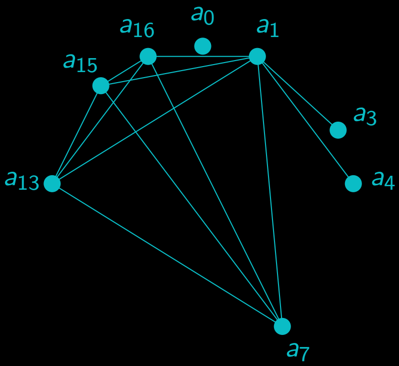




End-homogeneous sequence





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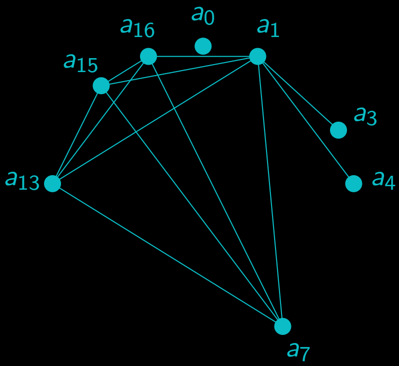
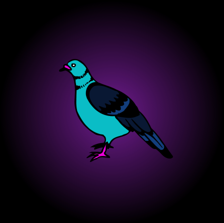
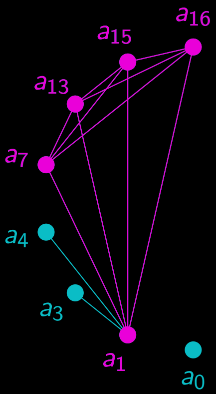
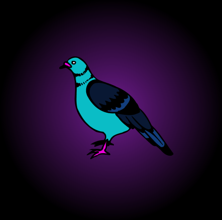


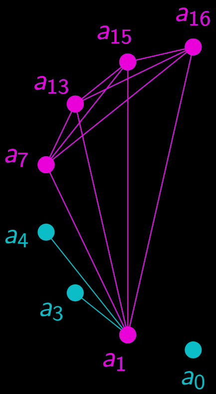
Homogeneous set

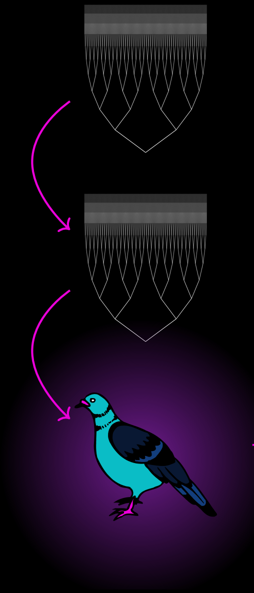




Homogeneous set

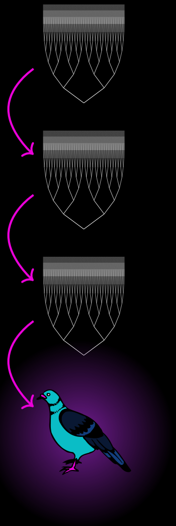




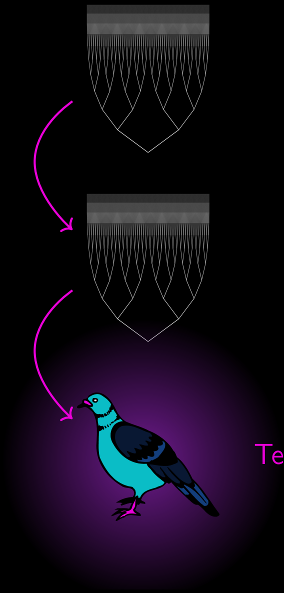


Ternary

Homogeneous set

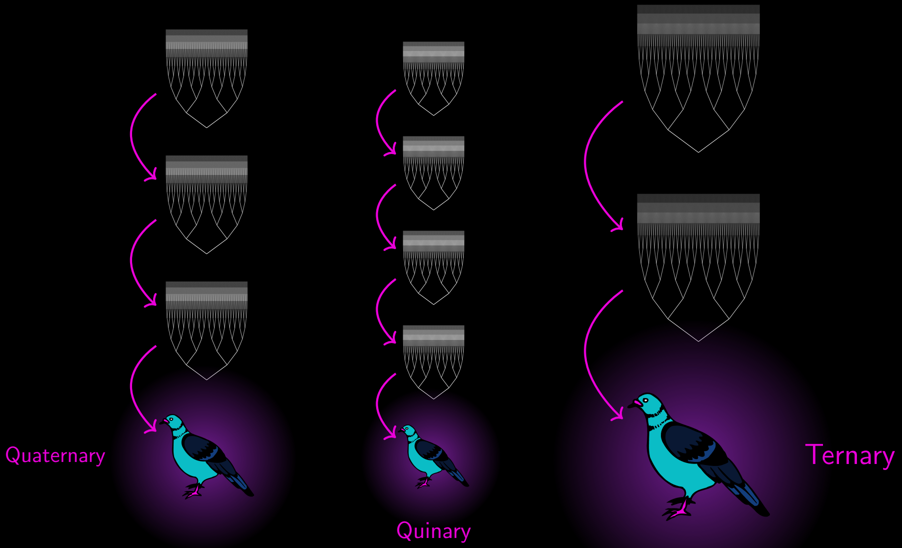


Quaternary

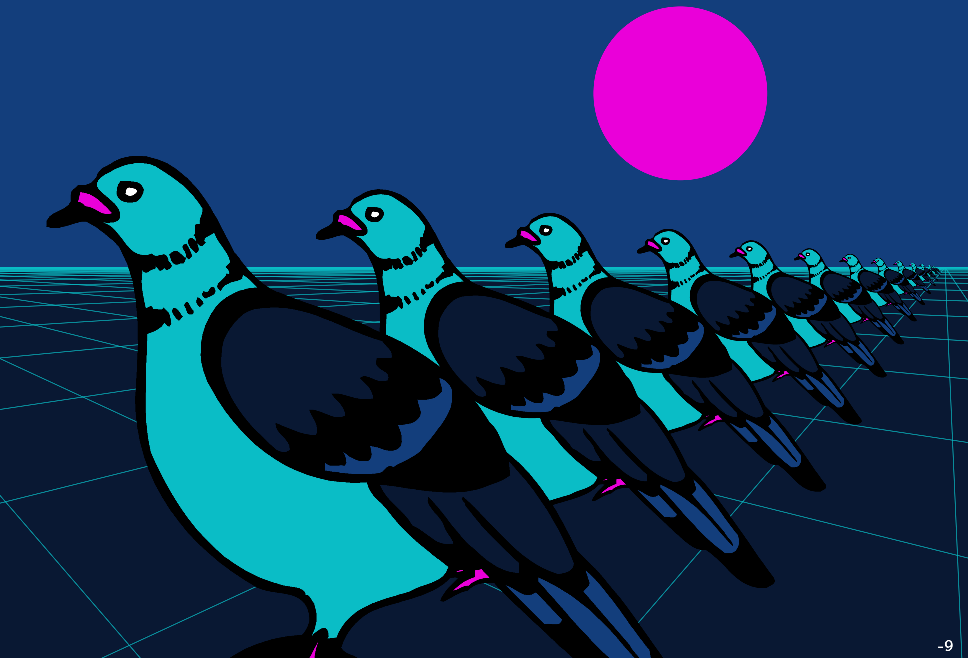


Ternary

Homogeneous set



You can't do this forever



What can you do?

What can you do?

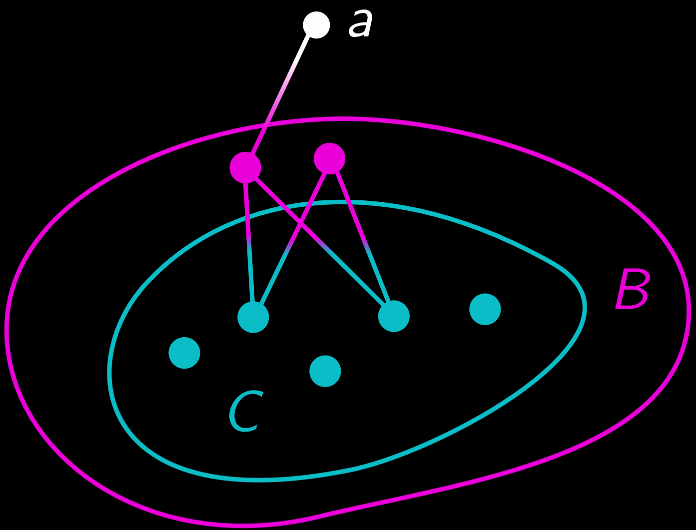
Large cardinal magic:
Erdős cardinals

What can you do?

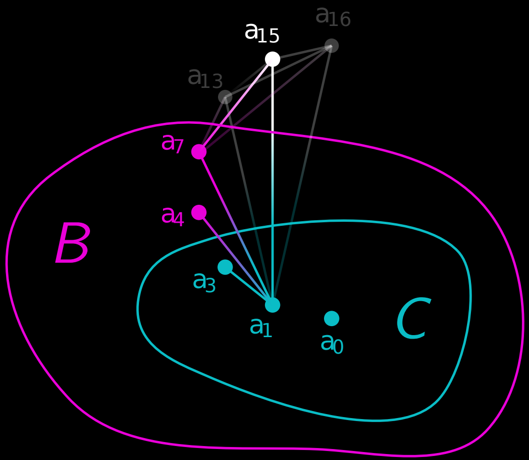
Large cardinal magic:
Erdős cardinals

Combinatorial tameness:
Stability

The type of a over B splits over C



The type of a_{15} over B splits over C



Stability

End-homogeneous + No splitting $\rightarrow$ Indiscernible

Stability

T is stable if

there is a κ such that for any type $p(x) \in S(B)$, there is a $C \subseteq B$ with $|C| \leq \kappa$ such that p does not split over C.

Stability

T is stable if

T is 'morally unary'.

Stability

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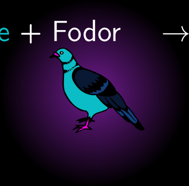
Stable + Fodor $\rightarrow$ every sufficiently long sequence has indiscernible subseqeunce

Stability

T is stable if

T is 'morally unary'.

Stable + Fodor $\longrightarrow$ every sufficiently long sequence has indiscernible subseqeunce



Higher-arity Fodor's lemma?

Pigeonhole : Ramsey :: Fodor : ???

Higher-arity Fodor's lemma?

Pigeonhole : Ramsey :: Fodor : k-ineffable

Higher-arity Fodor's lemma?

Erdős

$\vee$

completely ineffable

$\vee$

Pigeonhole : Ramsey :: Fodor : *k*-ineffable

$\vee$

ineffable

$\vee$

weakly compact

What can you do?

Large cardinal magic:
Erdős cardinals

Combinatorial tameness
of (hyper)graph

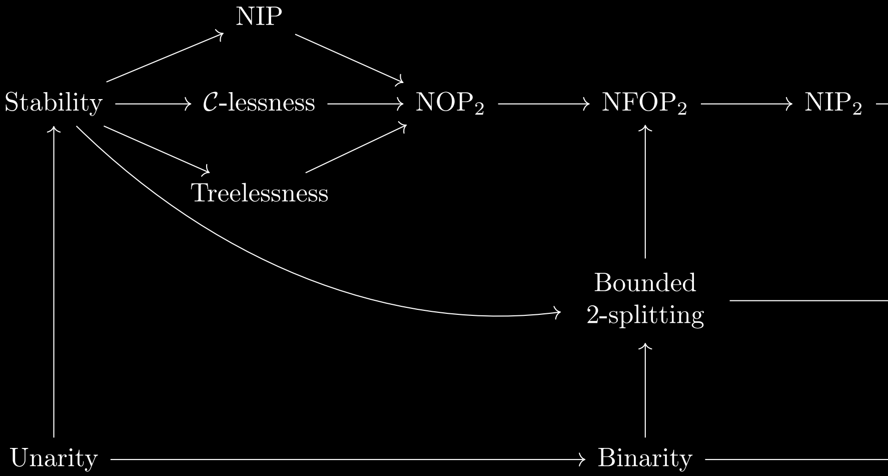
What can you do?

Large cardinal magic:
Erdős cardinals

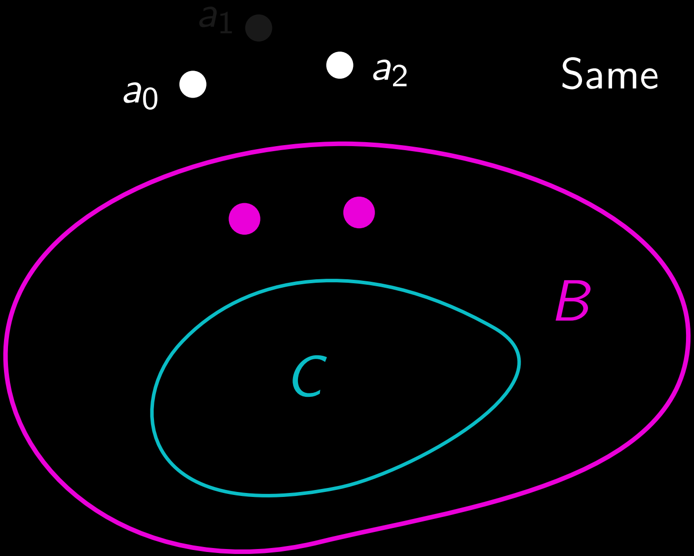
Combinatorial tameness
of (hyper)graph

Some combination of both?

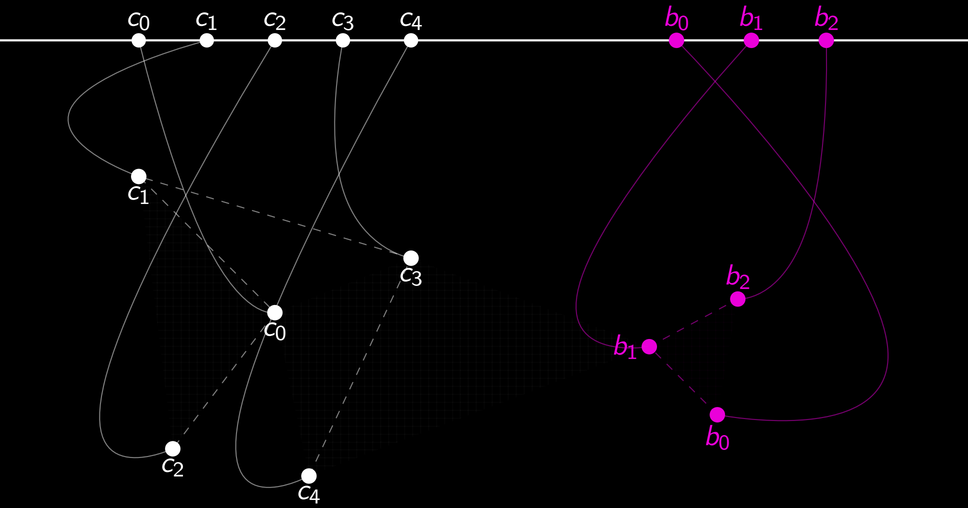
Higher-arity neostability



The type of $(a_0; a_1; a_2)$ over B 3-splits over C



A 3-end-homogeneous sequence



k-end-homogeneous + no k-splitting $\rightarrow$ indiscernible

Theorem

T has bounded k-splitting if

there is a κ such that for any type $p(x_0; x_1; \ldots; x_{k-1}) \in S(B)$, there is a $C \subseteq B$ with $|C| \leq \kappa$ such that p does not k-split over C.

Theorem

T has bounded k-splitting if

T is 'morally k-ary'.

Theorem

T has bounded k-splitting if

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Bounded k-splitting + μ k-ineffable $\rightarrow$ any sequence of length μ has indiscernible subsequence of length μ

Theorem

T has bounded k-splitting if

T is 'morally k-ary'.

Bounded k-splitting + μ k-ineffable $\rightarrow$ any sequence of length μ has indiscernible subsequence of length μ

Thank you